BOOK EIGHT

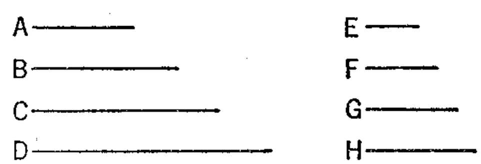
Proposition 1

If there be as many numbers as we please in continued proportion, and the extremes of them be prime to one another, the numbers are the least of those which have the same ratio with them.

Let there be as many numbers as we please, A, B, C, D, in continued proportion,

and let the extremes of them A, D be prime to one another;

I say that A, B, C, D are the least of those which have the same ratio with them.



For, if not, let E, F, G, H be less than A, B, C, D, and in the same ratio with them.

Now, since A, B, C, D are in the same ratio with E, F, G, H,

and the multitude of the numbers A, B, C, D is equal to the multitude of the numbers E, F, G, H,

therefore, *ex aequali*,

as A is to D, so is E to H. [VII. 14]

But A, D are prime,

primes are also least, [VII. 21]

and the least numbers measure those which have the same ratio the same number of times, the greater the greater and the less the less, that is, the antecedent the antecedent and the consequent the consequent. [VII. 20]

Therefore A measures E, the greater the less:

which is impossible.

Therefore E, F, G, H which are less than A, B, C, D are not in the same ratio with them.

Therefore A, B, C, D are the least of those which have the same ratio with them. Q. E. D.

Proposition 2

To find numbers in continued proportion, as many as may be prescribed, and the least that are in a given ratio.

Let the ratio of A to B be the given ratio in least numbers;

thus it is required to find numbers in continued proportion, as many as may be prescribed, and the least that are in the ratio of A to B.

Let four be prescribed;

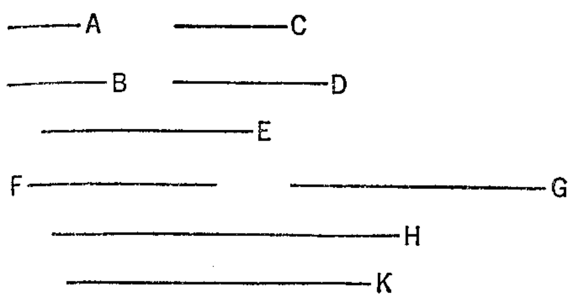
let A by multiplying itself make C, and by multiplying B let it make D;

let B by multiplying itself make E;

further, let A by multiplying C, D, E make F, G, H.

and let B by multiplying E make K.

Now, since A by multiplying itself has made C,



and by multiplying B has made D,

therefore, as A is to B, so is C to D. [VII. 17]

Again, since A by multiplying B has made D,

and B by multiplying itself has made E,

therefore the numbers A, B by multiplying B have made the numbers D, E respectively.

Therefore, as A is to B, so is D to E, [VII. 18]

But, as A is to B, so is C to D;

therefore also, as C is to D, so is D to E.

And, since A by multiplying C, D has made F, G,

therefore, as C is to D, so is F to G. [VII. 17]

But, as C is to D, so was A to B;

therefore also, as A is to B, so is F to G.

Again, since A by multiplying D, E has made G, H,

therefore, as D is to E, so is G to H. [VII. 17]

But, as D is to E, so is A to B.

Therefore also, as A is to B, so is G to H.

And, since A, B by multiplying E have made H, K,

therefore, as A is to B, so is H to K. [VII. 18]

But, as A is to B, so is F to G, and G to H.

Therefore also, as F is to G, so is G to H, and H to K;

therefore C, D, E, and F, G, H, K are proportional in the ratio of A to B.

I say next that they are the least numbers that are so.

For, since A, B are the least of those which have the same ratio with them, and the least of those which have the same ratio are prime to one another, [VII. 22]

therefore A, B are prime to one another.

And the numbers A, B by multiplying themselves respectively have made the numbers C, E, and by multiplying the numbers C, E respectively have made the numbers F, K;

therefore C, E and F, K are prime to one another respectively. [VII. 27]

But, if there be as many numbers as we please in continued proportion, and the extremes of them be prime to one another, they are the least of those which have the same ratio with them. [VIII. 1]

Therefore C, D, E and F, G, H, K are the least of those which have the same ratio with A, B.

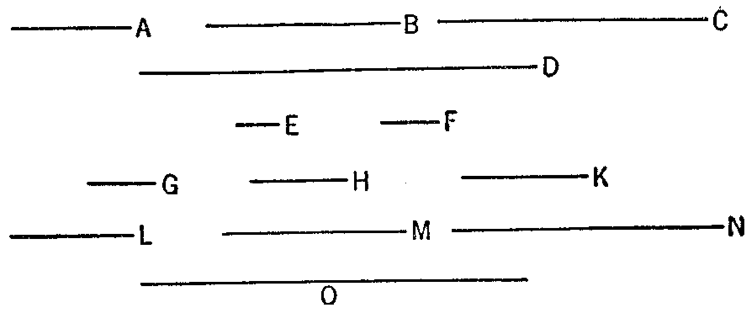
Q. E. D.

Porism. From this it is manifest that, if three numbers in continued proportion be the least of those which have the same ratio with them, the extremes of them are squares, and, if four numbers, cubes.

Proposition 3

If as many numbers as we please in continued proportion be the least of those which have the same ratio with them, the extremes of them are prime to one another.

Let as many numbers as we please, A, B, C, D, in continued proportion be the least of those which have the same ratio with them;



I say that the extremes of them A, D are prime to one another.

For let two numbers E, F, the least that are in the ratio of A, B, C, D, be taken, [VII. 33]

then three others G, H, K with the same property;

and others, more by one continually, [VIII. 2]

until the multitude taken becomes equal to the multitude of the numbers A, B, C, D.

Let them be taken, and let them be L, M, N, O.

Now, since E, F are the least of those which have the same ratio with them, they are prime to one another. [VII. 22]

And, since the numbers E, F by multiplying themselves respectively have made the numbers G, K, and by multiplying the numbers G, K respectively have made the numbers L, O, [VIII. 2, Por.]

therefore both G, K and L, O are prime to one another. [VII. 27]

And, since A, B, C, D are the least of those which have the same ratio with them,

while L, M, N, O are the least that are in the same ratio with A, B, C, D,

and the multitude of the numbers A, B, C, D is equal to the multitude of the numbers L, M, N, O,

therefore the numbers A, B, C, D are equal to the numbers L, M, N, O respectively;

therefore A is equal to L, and D to O.

And L, O are prime to one another.

Therefore A, D are also prime to one another. Q. E. D.

Proposition 4

Given as many ratios as we please in least numbers, to find numbers in continued proportion which are the least in the given ratios.

Let the given ratios in least numbers be that of A to B, that of C to D, and that of E to F;

thus it is required to find numbers in continued proportion which are the least that are in the ratio of A to B, in the ratio of C to D, and in the ratio of E to F.

Let G, the least number measured by B, C, be taken. [VII. 34]

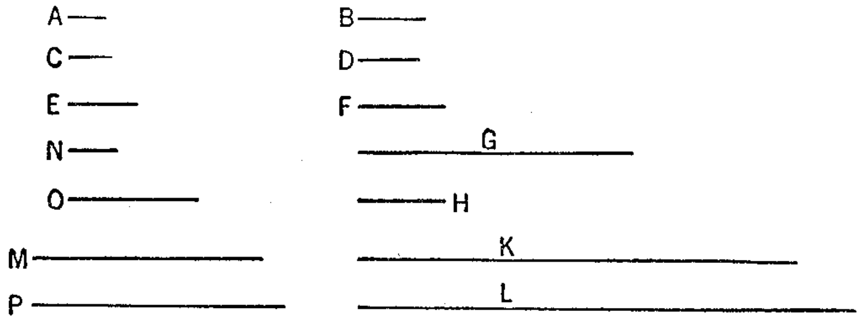
And, as many times as B measures G, so many times also let A measure H, and, as many times as C measures G, so many times also let D measure K.

Now E either measures or does not measure K.

First, let it measure it.

And, as many times as E measures K, so many times let F measure L also.

Now, since A measures H the same number of times that B measures G, therefore, as A is to B, so is H to G. [VII. Def. 20, VII. 13]



For the same reason also,

as C is to D, so is G to K,

and further, as E is to F, so is K to L;

therefore H, G, K, L are continuously proportional in the ratio of A to B, in the ratio of C to D, and in the ratio of E to F.

I say next that they are also the least that have this property.

For, if H, G, K, L are not the least numbers continuously proportional in the ratios of A to B, of C to D, and of E to F, let them be N, O, M, P.

Then since, as A is to B, so is N to O,

while A, B are least,

and the least numbers measure those which have the same ratio the same number of times, the greater the greater and the less the less, that is, the antecedent the antecedent and the consequent the consequent;

therefore B measures O. [VII. 20]

For the same reason

C also measures O;

therefore B, C measure O;

therefore the least number measured by B, C will also measure O. [VII. 35]

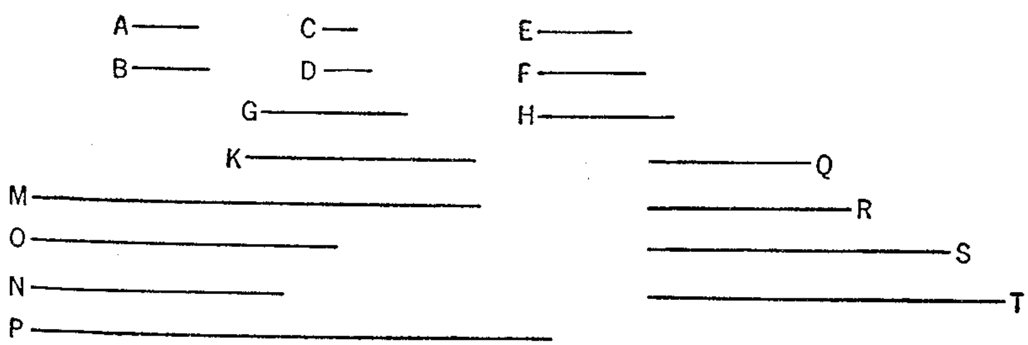
But G is the least number measured by B, C;

therefore G measures O, the greater the less:

which is impossible.

Therefore there will be no numbers less than H, G, K, L which are continuously in the ratio of A to B, of C to D, and of E to F.

Next, let E not measure K.



Let M, the least number measured by E, K, be taken.

And, as many times as K measures M, so many times let H, G measure N, O respectively, and, as many times as E measures M, so many times let F measure P also.

Since H measures N the same number of times that G measures O,

therefore, as H is to G, so is N to O. [VII. 13 and Def. 20]

But, as H is to G, so is A to B;

therefore also, as A is to B, so is N to O.

For the same reason also,

as C is to D, so is O to M.

Again, since E measures M the same number of times that F measures P,

therefore, as E is to F, so is M to P; [VII. 13 and Def. 20]

therefore N, O, M, P are continuously proportional in the ratios of A to B, of C to D, and of E to F.

I say next that they are also the least that are in the ratios $A:B$, $C:D$, $E:F$.

For, if not, there will be some numbers less than N, O, M, P continuously proportional in the ratios $A:B$, $C:D$, $E:F$.

Let them be Q, R, S, T.

Now since, as Q is to R, so is A to B.

while A, B are least,

and the least numbers measure those which have the same ratio with them the same number of times, the antecedent the antecedent and the consequent the consequent, [VII. 20]

therefore B measures R.

For the same reason C also measures R;

therefore B, C measure R.

Therefore the least number measured by B, C will also measure R. [VII. 35]

But G is the least number measured by B, C;

therefore G measures R.

And, as G is to R, so is K to S: [VII. 13]

therefore K also measures S.

But E also measures S;

therefore E, K measure S.

Therefore the least number measured by E, K will also measure S. [VII. 35]

But M is the least number measured by E, K;

therefore M measures S, the greater the less:

which is impossible.

Therefore there will not be any numbers less than N, O, M, P continuously proportional in the ratios of A to B, of C to D, and of E to F;

therefore N, O, M, P are the least numbers continuously proportional in the ratios $A:B$, $C:D$, $E:F$. Q. E. D.

PROPOSITION 5

Plane numbers have to one another the ratio compounded of the ratios of their sides.

Let A, B be plane numbers, and let the numbers C, D be the sides of A, and E, F of B;

I say that A has to B the ratio compounded of the ratios of the sides.

For, the ratios being given which C has to E and D to F, let the least numbers G, H, K that are continuously in the ratios $C:E$, $D:F$ be taken, so that,

as C is to E, so is G to H,

and, as D is to F, so is H to K. [VIII. 4]

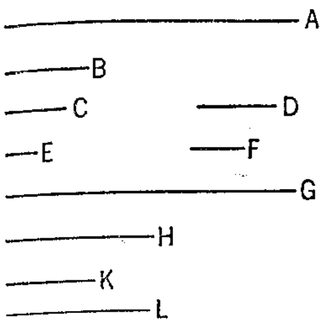
And let D by multiplying E make L.

Now, since D by multiplying C has made A, and by multiplying E has made L,

therefore, as C is to E, so is A to L. [VII. 17]

But, as C is to E, so is G to H;

therefore also, as G is to H, so is A to L.



Again, since E by multiplying D has made L, and further by multiplying F has made B,

therefore, as D is to F, so is L to B. [VII. 17]

But, as D is to F, so is H to K;

therefore also, as H is to K, so is L to B.

But it was also proved that,

as G is to H, so is A to L;

therefore, *ex aequali*,

as G is to K, so is A to B. [VII. 14]

But G has to K the ratio compounded of the ratios of the sides; therefore A also has to B the ratio compounded of the ratios of the sides.

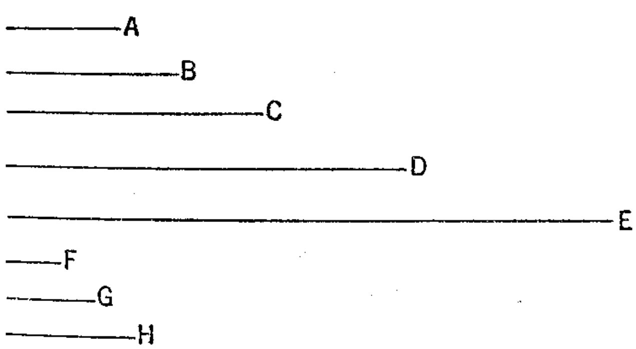
Q. E. D.

Proposition 6

If there be as many numbers as we please in continued proportion, and the first do not measure the second, neither will any other measure any other.

Let there be as many numbers as we please, A, B, C, D, E, in continued proportion, and let A not measure B;

I say that neither will any other measure any other.



Now it is manifest that A, B, C, D, E do not measure one another in order; for A does not even measure B.

I say, then, that neither will any other measure any other.

For, if possible, let A measure C.

And, however many A, B, C are, let as many numbers F, G, H, the least of those which have the same ratio with A, B, C, be taken. [VII. 33]

Now, since F, G, H are in the same ratio with A, B, C, and the multitude of the numbers A, B, C is equal to the multitude of the numbers F, G, H,

therefore, *ex aequali*, as A is to C, so is F to H. [VII. 14]

And since, as A is to B, so is F to G,

while A does not measure B,

therefore neither does F measure G; [VII. Def. 20]

therefore F is not an unit, for the unit measures any number.

Now F, H are prime to one another. [VIII. 3]

And, as F is to H, so is A to C;

therefore neither does A measure C.

Similarly we can prove that neither will any other measure any other.

Q. E. D.

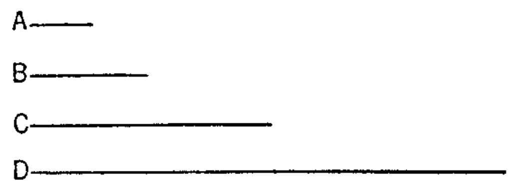
Proposition 7

If there be as many numbers as we please in continued proportion, and the first measure the last, it will measure the second also.

Let there be as many numbers as we please, A, B, C, D, in continued proportion; and let A measure D;

I say that A also measures B.

For, if A does not measure B, neither will any other of the numbers measure any other. [VIII. 6]



But A measures D.

Therefore A also measures B. Q. E. D.

Proposition 8

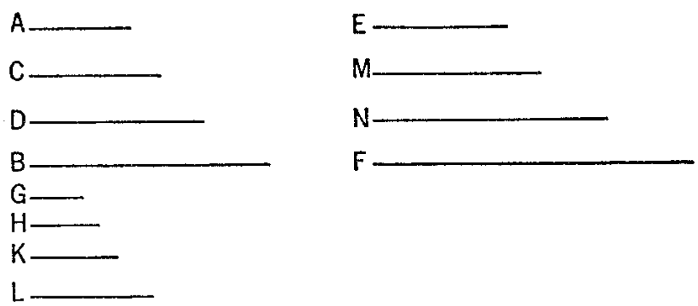
If between two numbers there fall numbers in continued proportion with them, then, however many numbers fall between them in continued proportion, so many will also fall in continued proportion between the numbers which have the same ratio with the original numbers.

Let the numbers C, D fall between the two numbers A, B in continued proportion with them, and let E be made in the same ratio to F as A is to B;

I say that, as many numbers as have fallen between A, B in continued proportion, so many will also fall between E, F in continued proportion.

For, as many as A, B, C, D are in multitude, let so many numbers G, H, K, L, the least of those which have the same ratio with A, C, D, B, be taken; [VII. 33]

therefore the extremes of them G, L are prime to one another. [VIII. 3]



Now, since A, C, D, B are in the same ratio with G, H, K, L, and the multitude of the numbers A, C, D, B is equal to the multitude of the numbers G, H, K, L,

therefore, *ex aequali*, as A is to B, so is G to L. [VII. 14]

But, as A is to B, so is E to F;

therefore also, as G is to L, so is E to F.

But G, L are prime,

primes are also least, [VII. 21]

and the least numbers measure those which have the same ratio the same number of times, the greater the greater and the less the less, that is, the antecedent the antecedent and the consequent the consequent. [VII. 20]

Therefore G measures E the same number of times as L measures F.

Next, as many times as G measures E, so many times let H, K also measure M, N respectively;

therefore G, H, K, L measure E, M, N, F the same number of times.

Therefore G, H, K, L are in the same ratio with E, M, N, F. [VII. Def. 20]

But G, H, K, L are in the same ratio with A, C, D, B;

therefore A, C, D, B are also in the same ratio with E, M, N, F.

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