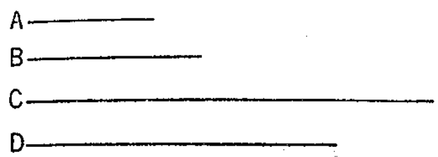
BOOK NINE

Proposition 1

If two similar plane numbers by multiplying one another make some number, the product will be square.

Let A, B be two similar plane numbers, and let A by multiplying B make C;



I say that C is square.

For let A by multiplying itself make D.

Therefore D is square.

Since then A by multiplying itself has made D, and by multiplying B has made C,

therefore, as A is to B, so is D to C. [VII. 17]

And, since A, B are similar plane numbers,

therefore one mean proportional number falls between A, B. [VIII. 18]

But, if numbers fall between two numbers in continued proportion, as many as fall between them, so many also fall between those which have the same ratio; [VIII. 8]

so that one mean proportional number falls between D, C also.

And D is square;

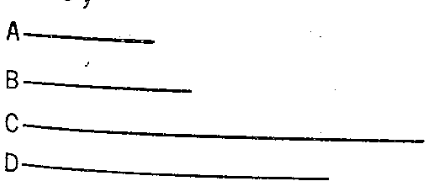
therefore C is also square. [VIII. 22]

Q. E. D.

Proposition 2

If two numbers by multiplying one another make a square number, they are similar plane numbers.

Let A, B be two numbers, and let A by multiplying B make the square number C;



I say that A, B are similar plane numbers.

For let A by multiplying itself make D;

therefore D is square.

Now, since A by multiplying itself has made D, and by multiplying B has made C,

therefore, as A is to B, so is D to C. [VII. 17]

And, since D is square, and C is so also,

therefore D, C are similar plane numbers.

Therefore one mean proportional number falls between D, C. [VIII. 18]

And, as D is to C, so is A to B;

therefore one mean proportional number falls between A, B also. [VIII. 8]

But, if one mean proportional number fall between two numbers, they are similar plane numbers; [VIII. 20]

therefore A, B are similar plane numbers. Q. E. D.

Proposition 3

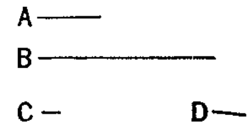
If a cube number by multiplying itself make some number, the product will be cube.

For let the cube number A by multiplying itself make B;

I say that B is cube.

For let C, the side of A, be taken, and let C by multiplying itself make D.

It is then manifest that C by multiplying D has made A.



Now, since C by multiplying itself has made D, therefore C measures D according to the units in itself.

But further the unit also measures C according to the units in it;

therefore, as the unit is to C, so is C to D. [VII. Def. 20]

Again, since C by multiplying D has made A,

therefore D measures A according to the units in C.

But the unit also measures C according to the units in it;

therefore, as the unit is to C, so is D to A.

But, as the unit is to C, so is C to D;

therefore also, as the unit is to C, so is C to D, and D to A.

Therefore between the unit and the number A two mean proportional numbers C, D have fallen in continued proportion.

Again, since A by multiplying itself has made B,

therefore A measures B according to the units in itself.

But the unit also measures A according to the units in it;

therefore, as the unit is to A, so is A to B. [VII. Def. 20]

But between the unit and A two mean proportional numbers have fallen; therefore two mean proportional numbers will also fall between A, B. [VIII. 8]

But, if two mean proportional numbers fall between two numbers, and the first be cube, the second will also be cube. [VIII. 23]

And A is cube;

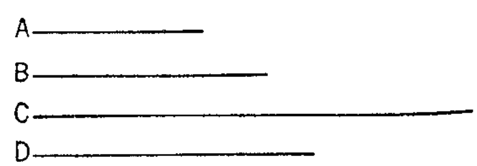
therefore B is also cube. Q. E. D.

Proposition 4

If a cube number by multiplying a cube number make some number, the product will be cube.

For let the cube number A by multiplying the cube number B make C;

I say that C is cube.



For let A by multiplying itself make D;

therefore D is cube. [IX. 3]

And, since A by multiplying itself has made D, and by multiplying B has made C

therefore, as A is to B, so is D to C. [VII. 17]

And, since A, B are cube numbers, A, B are similar solid numbers.

Therefore two mean proportional numbers fall between A, B; [VIII. 19] so that two mean proportional numbers will fall between D, C also. [VIII. 8]

And D is cube;

therefore C is also cube [VIII. 23]

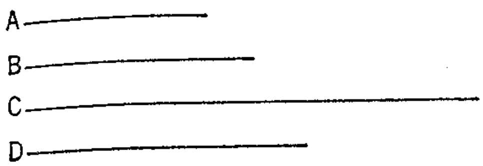
Q. E. D.

Proposition 5

If a cube number by multiplying any number make a cube number, the multiplied number will also be cube.

For let the cube number A by multiplying any number B make the cube number C;

I say that B is cube.



For let A by multiplying itself make D;

therefore D is cube. [IX. 3]

Now, since A by multiplying itself has made D, and by multiplying B has made C,

therefore, as A is to B, so is D to C. [VII. 17]

And since D, C are cube,

they are similar solid numbers.

Therefore two mean proportional numbers fall between D, C. [VIII. 19]

And, as D is to C, so is A to B;

therefore two mean proportional numbers fall between A, B also. [VIII. 8]

And A is cube;

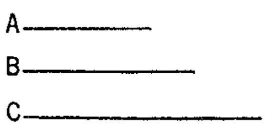
therefore B is also cube. [VIII. 23]

Proposition 6

If a number by multiplying itself make a cube number, it will itself also be cube.

For let the number A by multiplying itself make the cube number B;

I say that A is also cube.



For let A by multiplying B make C.

Since, then, A by multiplying itself has made B, and by multiplying B has made C,

therefore C is cube.

And, since A by multiplying itself has made B,

therefore A measures B according to the units in itself.

But the unit also measures A according to the units in it.

Therefore, as the unit is to A, so is A to B. [VII. Def. 20]

And, since A by multiplying B has made C,

therefore B measures C according to the units in A.

But the unit also measures A according to the units in it.

Therefore, as the unit is to A, so is B to C. [VII. Def. 20]

But, as the unit is to A, so is A to B;

therefore also, as A is to B, so is B to C.

And, since B, C are cube,

they are similar solid numbers.

Therefore there are two mean proportional numbers between B, C. [VIII. 19]

And, as B is to C, so is A to B.

Therefore there are two mean proportional numbers between A, B also. [VIII. 8]

And B is cube;

therefore A is also cube. [cf. VIII. 23]

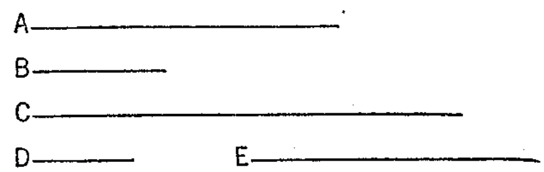
Q. E. D.

Proposition 7

If a composite number by multiplying any number make some number, the product will be solid.

For, let the composite number A by multiplying any number B make C;

I say that C is solid.



For, since A is composite, it will be measured by some number. [VII. Def. 13]

Let it be measured by D;

and, as many times as D measures A, so many units let there be in E.

Since, then, D measures A according to the units in E,

therefore E by multiplying D has made A. [VII. Def. 15]

And, since A by multiplying B has made C,

and A is the product of D, E,

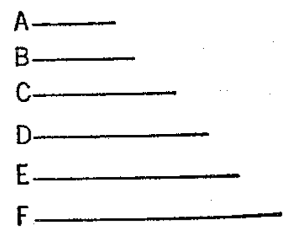
therefore the product of D, E by multiplying B has made C.

Therefore C is solid, and D, E, B are its sides. Q. E. D.

Proposition 8

If as many numbers as we please beginning from an unit be in continued proportion, the third from the unit will be square, as will also those which successively leave out one; the fourth will be cube, as will also all those which leave out two; and the seventh will be at once cube and square, as will also those which leave out five.

Let there be as many numbers as we please, A, B, C, D, E, F, beginning from an unit and in continued proportion;



I say that B, the third from the unit, is square, as are also all those which leave out one; C, the fourth, is cube, as are also all those which leave out two; and F, the seventh, is at once cube and square, as are also all those which leave out five.

For since, as the unit is to A, so is A to B,

therefore the unit measures the number A the same number of times that A measures B. [VII. Def. 20]

But the unit measures the number A according to the units in it;

therefore A also measures B according to the units in A.

Therefore A by multiplying itself has made B;

therefore B is square.

And, since B, C, D are in continued proportion, and B is square,

therefore D is also square. [VIII. 22]

For the same reason

F is also square.

Similarly we can prove that all those which leave out one are square.

I say next that C, the fourth from the unit, is cube, as are also all those which leave out two.

For since, as the unit is to A, so is B to C,

therefore the unit measures the number A the same number of times that B measures C.

But the unit measures the number A according to the units in A;

therefore B also measures C according to the units in A.

Therefore A by multiplying B has made C.

Since then A by multiplying itself has made B, and by multiplying B has made C,

therefore C is cube.

And, since C, D, E, F are in continued proportion, and C is cube,

therefore F is also cube. [VIII. 23]

But it was also proved square;

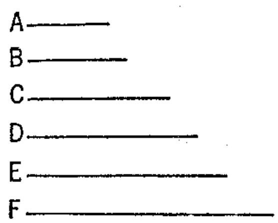
therefore the seventh from the unit is both cube and square.

Similarly we can prove that all the numbers which leave out five are also both cube and square. Q. E. D.

Proposition 9

If as many numbers as we please beginning from an unit be in continued proportion, and the number after the unit be square, all the rest will also be square. And, if the number after the unit be cube, all the rest will also be cube.

Let there be as many numbers as we please, A, B, C, D, E, F, beginning from an unit and in continued proportion, and let A, the number after the unit, be square;



I say that all the rest will also be square.

Now it has been proved that B, the third from the unit, is square, as are also all those which leave out one; [IX. 8]

I say that all the rest are also square.

For, since A, B, C are in continued proportion,

and A is square,

therefore C is also square. [VIII. 22]

Again, since B, C, D are in continued proportion,

and B is square,

D is also square. [VIII. 22]

Similarly we can prove that all the rest are also square.

Next, let A be cube;

I say that all the rest are also cube.

Now it has been proved that C, the fourth from the unit, is cube, as also are all those which leave out two; [IX. 8]

I say that all the rest are also cube.

For, since, as the unit is to A, so is A to B,

therefore the unit measures A the same number of times as A measures B.

But the unit measures A according to the units in it;

therefore A also measures B according to the units in itself;

therefore A by multiplying itself has made B.

And A is cube.

But, if a cube number by multiplying itself make some number, the product is cube. [IX. 3]

Therefore B is also cube.

And, since the four numbers A, B, C, D are in continued proportion,

and A is cube,

D also is cube. [VIII. 23]

For the same reason

E is also cube, and similarly all the rest are cube. Q. E. D.

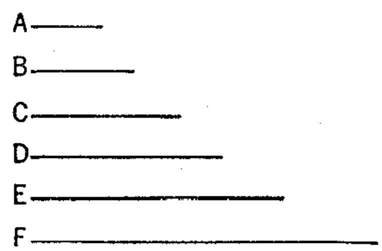
Proposition 10

If as many numbers as we please beginning from an unit be in continued proportion, and the number after the unit be not square, neither will any other be square except the third from the unit and all those which leave out one. And, if the number after the unit be not cube, neither will any other be cube except the fourth from the unit and all those which leave out two.

Let there be as many numbers as we please, *A*, *B*, *C*, *D*, *E*, *F*, beginning from an unit and in continued proportion,

and let *A*, the number after the unit, not be square;

I say that neither will any other be square except the third from the unit <and those which leave out one>.



For, if possible, let *C* be square.

But *B* is also square; [IX. 8]

[therefore *B*, *C* have to one another the ratio which a square number has to a square number].

And, as *B* is to *C*, so is *A* to *B*;

therefore *A*, *B* have to one another the ratio which a square number has to a square number;

[so that *A*, *B* are similar plane numbers]. [VIII. 26, converse]

And *B* is square;

therefore *A* is also square:

which is contrary to the hypothesis.

Therefore *C* is not square.

Similarly we can prove that neither is any other of the numbers square except the third from the unit and those which leave out one.

Next, let *A* not be cube.

I say that neither will any other be cube except the fourth from the unit and those which leave out two.

For, if possible, let *D* be cube.

Now *C* is also cube; for it is fourth from the unit. [IX. 8]

And, as *C* is to *D*, so is *B* to *C*;

therefore *B* also has to *C* the ratio which a cube has to a cube.

And *C* is cube;

therefore *B* is also cube. [VIII. 25]

And since, as the unit is to *A*, so is *A* to *B*,

and the unit measures *A* according to the units in it,

therefore *A* also measures *B* according to the units in itself;

therefore *A* by multiplying itself has made the cube number *B*.

But, if a number by multiplying itself make a cube number, it is also itself cube. [IX. 6]

Therefore *A* is also cube:

which is contrary to the hypothesis.

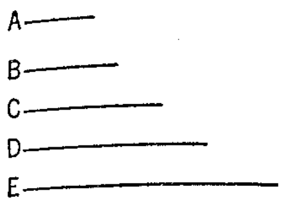
Therefore *D* is not cube.

Similarly we can prove that neither is any other of the numbers cube except the fourth from the unit and those which leave out two. Q. E. D.

Proposition 11

If as many numbers as we please beginning from an unit be in continued proportion, the less measures the greater according to some one of the numbers which have place among the proportional numbers.

Let there be as many numbers as we please, *B*, *C*, *D*, *E*, beginning from the unit *A* and in continued proportion;



I say that *B*, the least of the numbers *B*, *C*, *D*, *E*, measures *E* according to some one of the numbers *C*, *D*.

For since, as the unit *A* is to *B*, so is *D* to *E*, therefore the unit *A* measures the number *B* the same number of times as *D* measures *E*;

therefore, alternately, the unit *A* measures *D* the same number of times as *B* measures *E*. [VII. 15]

But the unit *A* measures *D* according to the units in it;

therefore *B* also measures *E* according to the units in *D*;

so that *B* the less measures *E* the greater according to some number of those which have place among the proportional numbers.—

Porism. And it is manifest that, whatever place the measuring number has, reckoned from the unit, the same place also has the number according to which it measures, reckoned from the number measured, in the direction of the number before it.— Q. E. D.

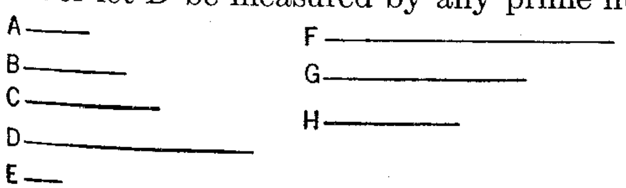
Proposition 12

If as many numbers as we please beginning from an unit be in continued proportion, by however many prime numbers the last is measured, the next to the unit will also be measured by the same.

Let there be as many numbers as we please, *A*, *B*, *C*, *D*, beginning from an unit, and in continued proportion;

I say that, by however many prime numbers *D* is measured, *A* will also be measured by the same.

For let *D* be measured by any prime number *E*;



I say that *E* measures *A*.

For suppose it does not;

now *E* is prime, and any prime number is prime to any which it does not measure; [VII. 29]

therefore *E*, *A* are prime to one another.

And, since *E* measures *D*, let it measure it according to *F*,

therefore *E* by multiplying *F* has made *D*.

Again, since *A* measures *D* according to the units in *C*, [IX. 11 and Por.]

therefore *A* by multiplying *C* has made *D*.

But, further, *E* has also by multiplying *F* made *D*;

therefore the product of *A*, *C* is equal to the product of *E*, *F*.

Therefore, as *A* is to *E*, so is *F* to *C*. [VII. 19]

But *A*, *E* are prime,

primes are also least, [VII. 21]

and the least measure those which have the same ratio the same number of times, the antecedent the antecedent and the consequent the consequent;

[VII. 20]

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